UNIT-1

(Lecture-4)

Realization of Digital Systems:

Cascade Realization of an IIR Systems

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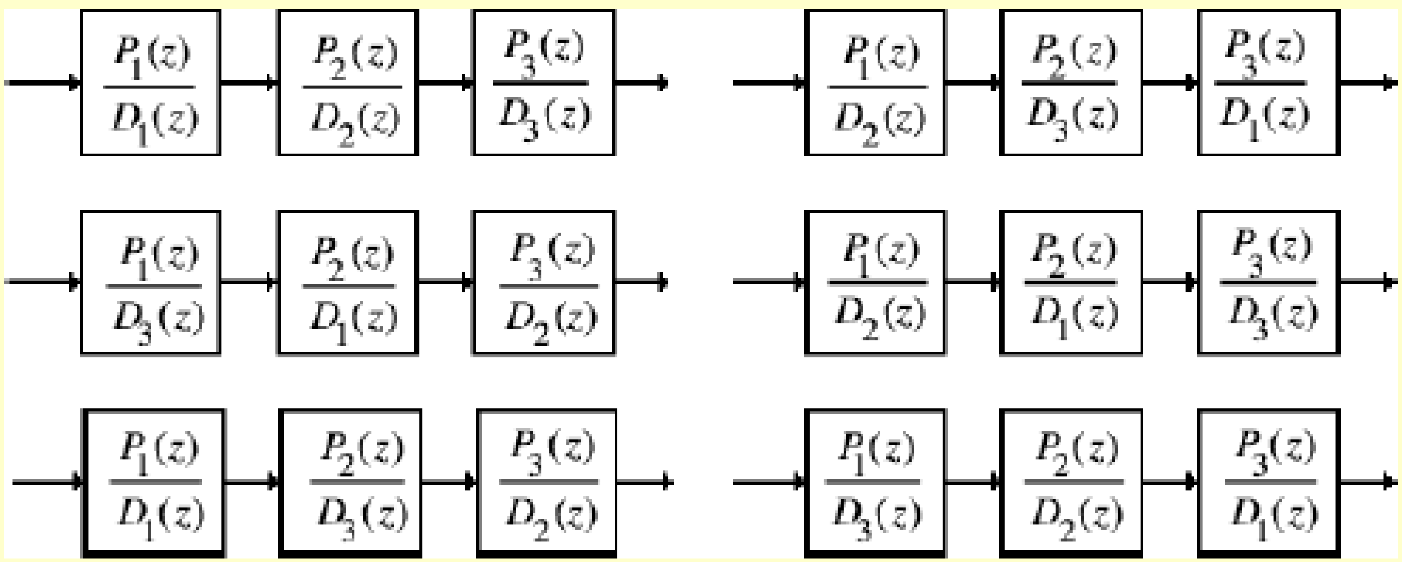
Cascade Form IIR Digital Filter Structures

- By expressing the numerator and the denominator polynomials of the transfer function as a product of polynomials of lower degree, a digital filter can be realized as a cascade of low-order filter sections
- Consider, for example, $H(z) = P(z)/D(z)$ expressed as

$$H(z) = \frac{P(z)}{D(z)} = \frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$

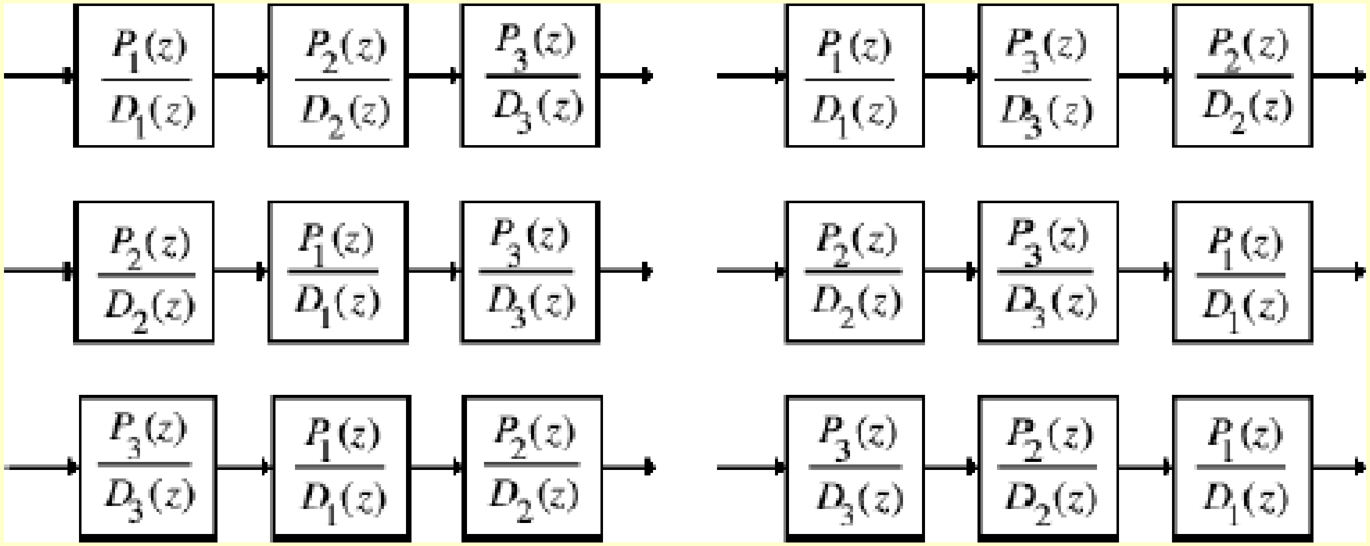
Cascade Form IIR Digital Filter Structures

- Examples of cascade realizations obtained by different pole-zero pairings are shown below



Cascade Form IIR Digital Filter Structures

- Examples of cascade realizations obtained by different ordering of sections are shown below



Cascade Form IIR Digital Filter Structures

- There are altogether a total of 36 different cascade realizations of

$$H(z) = \frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$

based on different pole-zero-pairings and different orderings

- Due to finite wordlength effects, each such cascade realization behaves differently from others

Cascade Form IIR Digital Filter Structures

- Usually, the polynomials are factored into a product of 1st-order and 2nd-order polynomials
- In this case $H(z)$ is expressed as

$$H(z) = p_0 \prod_k \left(\frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

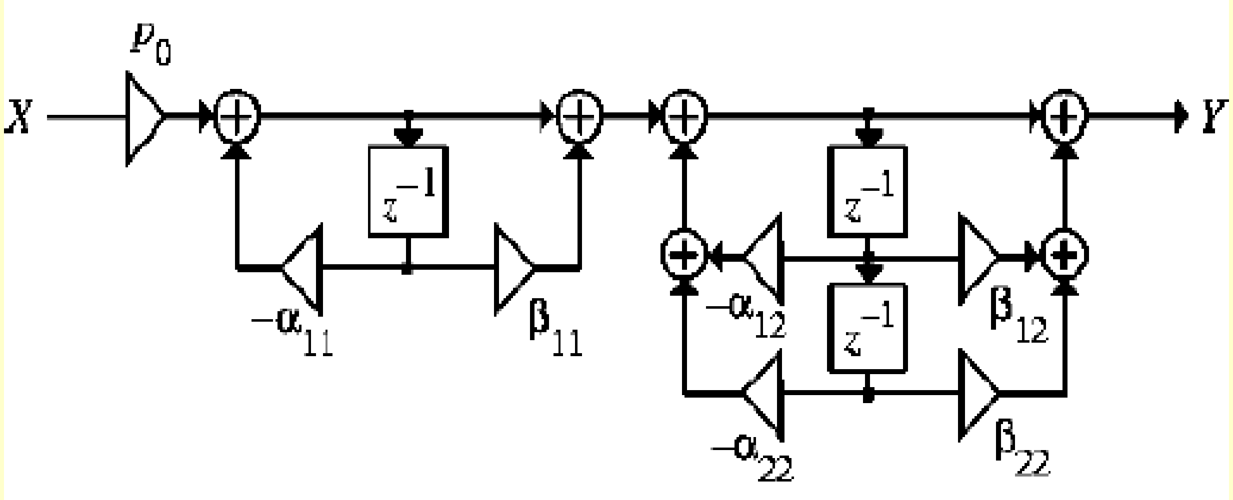
- In the above, for a first-order factor $\alpha_{2k} = \beta_{2k} = 0$

Cascade Form IIR Digital Filter Structures

- Consider the 3rd-order transfer function

$$H(z) = p_0 \left(\frac{1 + \beta_{11} z^{-1}}{1 + \alpha_{11} z^{-1}} \right) \left(\frac{1 + \beta_{12} z^{-1} + \beta_{22} z^{-2}}{1 + \alpha_{12} z^{-1} + \alpha_{22} z^{-2}} \right)$$

- One possible realization is shown below



Cascade Form IIR Digital Filter Structures

- Example: Direct form II and cascade form realizations of

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

$$= \left(\frac{0.44 + 0.362z^{-1} + 0.02z^{-2}}{1 + 0.8z^{-1} + 0.5z^{-2}} \right) \left(\frac{z^{-1}}{1 - 0.4z^{-1}} \right)$$

are shown on the next slide

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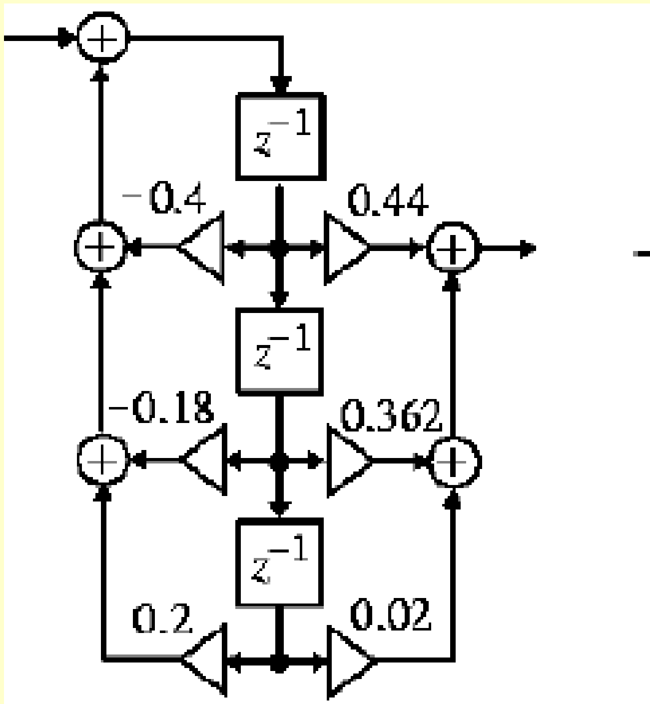
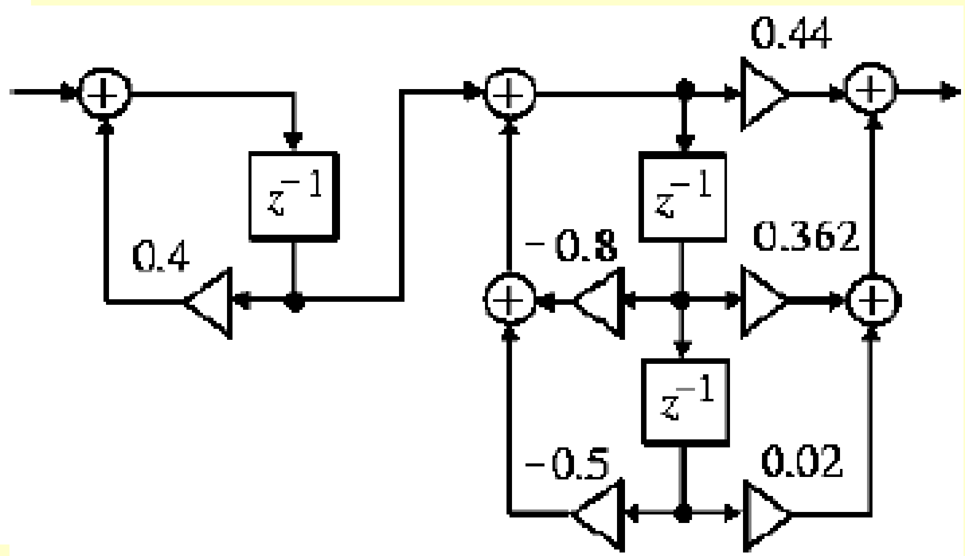
Cascade Form IIR Digital Filter Structures



Direct form II



Cascade form